

Series 03

Exercise 01 : Say whether the following functions are holomorphic in D

1. $g(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}, \quad D = \mathbb{C}^*,$
2. $g(z) = e^{2x}(\cos 2y + i(\sin 2y)) - iy + x, \quad D = \mathbb{C},$
3. $g(z) = e^{x^2-y^2}\cos(2xy) + ie^{x^2-y^2}\sin(2xy), \quad D = \mathbb{C},$
4. $g(z) = x^2 - y^2 - 2ixy + 2x + 2iy, \quad D = \mathbb{C},$
5. $g(z) = e^z, \quad D = \mathbb{C},$
6. $g(z) = \sin(z), \quad D = \mathbb{C}.$

Exercise 02 :

Show that if $f(z) = u + iv$ is holomorphic then u and v are harmonic.

Exercise 03 : Find holomorphic functions $f(z) = u + iv$ in the following cases:

1. $v(x, y) = xy^2 - \frac{1}{3}x^3$
2. $u(x, y) = 2x^3 - 6xy^2 + x^2 - y^2 - y$